An Insurance Mechanism for Public Goods Under Uncertainty*

Stephen Aultman,† Jay S. Coggins,‡,§ and Terrence M. Hurley‡

Draft: December 4, 2013

Abstract

Achieving an efficient level of one or more public goods is challenging from an incentive standpoint. Agents will lie when asked to report their preferences, which must typically be known in order for a planner to compute the optimum. The problem of public goods in the presence of uncertainty is challenging in another way because efficiency requires that risk and the cost of the public good must be shared optimally between agents and across states. We combine a public-insurance plan with the decision about a public good and show that under certain conditions all agents will prefer the package of insurance and the public good to insurance alone. Therefore the package is “insurance individually rational.” The insurance component of the package ensures full insurance for all agents, which in turn renders risk preferences irrelevant. In our framework a planner can implement the set of Pareto-efficient and insurance individually rational public projects in undominated strategies.

Keywords: Public goods; uncertainty; insurance

---

*We thank Steve Polasky, Hamid Mohtadi, and seminar participants at the University of Minnesota for helpful discussions.
†USAA Insurance, Inc., San Antonio TX
‡Department of Applied Economics, University of Minnesota, 1994 Buford Avenue, St. Paul MN 55108
§Corresponding author: jcoggins@umn.edu
1 Introduction

The new manager of a flood-prone city wishes to improve the well-being of her residents. Addressing the threat posed each spring by the nearby river stands atop the agenda. Two proposals have been mooted in the city council. The first is to build a protective levee, which would reduce the probability of flood damage. The levee is a public good, though its effect is mediated by the exogenous risk imposed by nature. The second is to provide flood insurance. This plan calls for the manager to solicit individual applications from homeowners, to bundle them together, and to secure reinsurance from an outside reinsurance market. Private individual flood insurance is rarely available, and is expensive when it is. Because the price of insuring a large portfolio is relatively low, the insurance program too is in the nature of a public good.

An interesting question is which proposal, levee or insurance, should be adopted. Our answer is: both. The levee, by reducing the probability of damage, also reduces the cost of insurance. Insurance, in turn, at least for risk-averse agents, increases expected utility and thus also improves well-being. But a joint levee-insurance program, coordinated so that the two components are designed optimally and in concert, provides a welfare surplus that is even greater.

Another question, also interesting, is just how the manager is to craft and execute the joint program. Selecting the optimal level of the public good, and inducing agents to participate optimally, are both challenging. An advisor might be tempted to suggest employing Samuelson’s (1954) condition, which identifies the efficient levels of public and private goods. But the condition requires the assumption that the planner knows individual preferences. It also ignores uncertainty.

With exogenous uncertainty of the kind we examine, Samuelson’s condition is not directly applicable. A counterpart to Samuelson for this situation is available, however. Graham (1992) has shown how to identify the set of Pareto-efficient vectors of public goods. He derives a net-benefit criterion that, in addition to selecting the quantity of public goods to be provided, also selects an associated vector of state-contingent payments that share risk optimally among agents and across states. The contingent payments simultaneously fund the provision of the public goods and complete a missing market for risk.

Though powerful, the Graham approach does present difficulties. Being a general-equilibrium model, his framework applies only to a closed economy. For small local problems, where computing the optimal vector of payments might be feasible computationally, this can be a limitation if the risk facing the economy is a natural disaster or some other
collective risk. The public good must be funded locally even in the worst state, where all agents have suffered losses. Though Graham’s definition of efficiency will be met, the level of public good might be quite limited. In practice, such risks are shared as widely as possible, typically through public or private insurance.

By assuming that preferences are known, Graham also ignores incentives. The Samuelson condition is silent on the question of incentives, but the 1954 paper is not. Samuelson understood well, and articulated clearly, that his condition cannot be achieved by a market scheme.\textsuperscript{1} The literature tracing back to this insight is now quite large. It is also mostly negative. Hurwicz and Walker (1990, p. 683), for example, write, “It has been shown by Hurwicz (1975), Green and Laffont (1979), and Walker (1980) that no mechanism for deciding upon the provision and financing of public goods can always yield Pareto optimal outcomes if truthful behavior is always a dominant strategy for each of the mechanism’s participants.” See also Zhou (1991); Barberà and Jackson (1994); and Moreno and Moscoso (2013).

We have looked in vain for a mechanism that addresses incentives in the provision of public goods under uncertainty of the Graham type. The purpose of the present paper is to propose and analyze such a mechanism. We build upon Graham’s foundation, extending it in two ways. First, we offer a practical method for selecting a specific element of the efficient frontier.\textsuperscript{2} Second, we address explicitly the question of incentives by devising an insurance-based mechanism that, while balancing the budget, fully implements the set of Pareto-efficient and individually rational insurance projects in undominated strategies.

The project of mechanism design involves inducing agents to reveal their preferences, or their valuation of a public project, which are unknown to the planner. We take a different approach, one that is made possible by the presence of uncertainty. We introduce an exogenous reinsurance market, treating the Graham risk-sharing payments explicitly as an insurance plan. The idea is not to discover risk preferences, but rather to render them irrelevant. Our planner offers risk-averse agents fair insurance. Their response is to insure fully and thus to become indifferent as to which state occurs.\textsuperscript{3}

\textsuperscript{1}Samuelson writes, “However no decentralized pricing system can serve to determine optimally these levels of collective consumption. Other kinds of “voting” or “signaling” would have to be tried. But, and this is the point sensed by Wicksell but perhaps not fully appreciated by Lindahl, now it is in the selfish interest of each person to give false signals, to pretend to have less interest in a given collective consumption activity than he really has, etc.” (1954, p. 388, emphasis in original).

\textsuperscript{2}Chavas and Coggins (2003) adapt the notion of egalitarian equivalence (Pazner and Schmeidler 1978) to refine the Graham criterion so as to identify a subset of the Pareto-efficient outcomes having attractive equity properties.

\textsuperscript{3}This idea has an echo in the replicating portfolio employed by Black and Scholes (1973) and Merton
As in Graham, the payments serve both to fund a public good and to share risk optimally. In our framework, though, risk is shared both between agents in the model and between the model economy and a large exogenous capital market. Our risk-sharing payments differ from Graham’s also in that ours take a particular form, consisting of a state-independent “two-part premium” together with an indemnification payment in the event of loss. The two-part premium payments are in the nature of insurance payments, but a pre-defined portion is used to pay for the public good. All decisions, the level of the public good and the state-contingent payment scheme, are taken *ex ante*.

In our model there are two states of the world and an arbitrary number of risk-averse agents. State 1 is the bad state in that aggregate losses are positive, which means that the economy faces aggregate risk. Each agent suffers loss in state 1, but loss can be negative for those who gain in the bad state. The planner has complete information regarding each agent’s loss, but individual risk preferences and endowed wealth are private information.\(^4\) A single public good, with well-behaved cost function, has the effect of reducing the probability of the bad state. Enough regularity is imposed to ensure the existence of a unique optimal level of the public good.

The device employed by the planner to select a particular element of the set of efficient projects is a rule for sharing the cost of the public good. Cost shares are required to sum to one, thus ensuring that the public good is affordable in both states. The economic gain conferred by the public good in our framework can be measured by the resulting reduction in the aggregate cost of insurance, relative to the situation with no public good. At the optimum, this benefit exceeds the associated cost and thereby creates an economic surplus. We show how the surplus can be divided among agents in such a way that each is strictly better off than at the endowed lottery. The combination of the public good and the two-part premium payments is therefore both efficient and individually rational.

Our mechanism is built up from this foundation. The planner, a player in the game form, offers each agent a take-it-or-leave-it package, consisting of a two-part premium payment and the optimal public good. Each agent’s strategy space is binary: she either accepts the package or declines it. The project is adopted only if everyone accepts, and the main result

\(^{1973}\) There too, an agent who maintains the correct portfolio is fully insured and so risk preferences are irrelevant.

\(^4\) Cabrales et al. (2003) argue that losses, in the event of a fire, are very difficult for an agency or even neighbors to estimate. We are less sure. Private insurers, after all, have a keen interest in understanding their financial exposure should a home be damaged by flood or fire, and also in verifying losses when they occur. Insurance companies devote considerable resources to predicting and assessing damages, activities that may be expected to face court scrutiny.
of the paper establishes that accepting is an undominated strategy. Our use of insurance as a way to solve the incentive problem appears to be new. Our assumption that the planner knows each agent’s damages in the bad state is not innocuous, though. It appears, but we have not yet shown, that even with insurance our agents would not truthfully reveal the value of their damages. Thus, the result of Cabrales et al. (2003) appears to survive in the case in which insurance payments also fund a public good.

2 The economic environment

Consider an uncertain economy with n agents, indexed by \( i \in I = \{1, \ldots, n\} \), and two possible states of the world, indexed by \( j = 1, 2 \). State 1 is designated the “bad state” from society’s perspective. The economy is endowed with an initial probability \( \bar{p} \in (0, 1) \) of state 1 occurring. Goods are money in each state, which is private, and a single public good. Each agent has a state-independent monetary endowment of \( w_i > 0 \). In state 1 agent \( i \) suffers loss \( D_i \), possibly negative (if \( i \)'s financial position is improved in state 1), but never greater than her endowment: \( D_i \leq w_i \). Because they have no stake in the public good, we ignore agents for whom \( D_i = 0 \).

Each agent’s consumption set, vectors of dollars in the two states, is \( \mathbb{R}^2 \). The vector \( \mathbf{e}_i = (w_i - D_i, w_i) \) denotes \( i \)'s endowed vector of state-dependent wealth, with \( e^j = \sum_i e^j_i \) and \( \mathbf{e} = (\mathbf{e}_1, \ldots, \mathbf{e}_n) \). Denote \( i \)'s vector of realized wealth in the two states by \( \mathbf{\gamma}_i = (\gamma^1_i, \gamma^2_i) \), \( \mathbf{\gamma} = (\gamma^1, \ldots, \gamma_n) \) is an allocation. The sense in which state 1 is bad is that aggregate damages, \( D = \sum_i D_i \), are strictly positive. Agent \( i \)'s preferences for wealth are given by the twice-differentiable risk-averse von Neumann-Morgenstern utility function \( u_i : \mathbb{R} \to \mathbb{R} \), with \( u'(\cdot) > 0 \) and \( u''(\cdot) < 0 \).

A public agency or planner has complete information regarding the vector of damages \( \mathbf{D} = (D_1, \ldots, D_n) \). The \( w_i \) and the \( u_i \) are private, known only to agent \( i \). The planner has exclusive access to a technology for producing a public good \( Z \in \mathbb{R}_+ \) that has the effect of reducing the probability of state 1 occurring.\(^5\) The public good affects probabilities according to the twice-differentiable function \( p : \mathbb{R}_+ \to [0, 1] \), with \( p(0) = \bar{p} \), \( p'(Z) < 0 \), and \( p''(Z) > 0 \). A lottery for \( i \) is \( (\gamma_i, p(Z)) \), a vector of realized wealth together with a probability, and \( i \)'s endowed lottery is \( (\mathbf{e}_i, \bar{p}) \). If costs are ignored, expected utility is strictly increasing in \( Z \) for \( i \) with \( D_i > 0 \) and strictly decreasing in \( Z \) for \( i \) with \( D_i < 0 \).

\(^5\)Our public good provides public protection. It would also be interesting to consider the possibility that \( Z \) affects losses, the \( D_i \), while leaving the probability of loss unchanged. This question is deferred to later work.
The technology is characterized by its dual, the twice-differentiable, strictly increasing, and strictly convex cost function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, with $c(0) = 0$. The cost of the public good is assumed to be independent of the state. To guarantee that the problem is interesting, we assume also that $-p'(0) \sum_i D_i > c'(0)$. Without this assumption the optimal $Z$ is zero.

Agents’ state-contingent payments serve two functions. They are used to fund $Z$ and to share risk between agents and across states and also between the economy and a larger external capital market. Let $t_i = (t^1_i, t^2_i) \in \mathbb{R}^2$ be the vector of transfers paid by $i$, negative if payment is received, and let $t = (t_1, \ldots, t_n)$. Wealth realized by $i$ is $w_i - D_i - t^1_i$ in state 1 and $w_i - t^2_i$ in state 2. A public project is $(t, Z)$. Given a public project, expected utility for $i$ is

$$\tilde{u}_i(t_i, Z; e_i) = p(Z) u_i(w_i - D_i - t^1_i) + (1 - p(Z)) u_i(w_i - t^2_i).$$

The planner has exclusive access to an exogenous reinsurance market and can sign a risk-sharing contract with that market on behalf of agents. By tapping the reinsurance market, the planner is able to provide indemnification benefits to agents. Insurance imposes a particular form upon the state-dependent payments. A state-independent premium, $a_i$, must be paid in both states and an indemnity benefit, $b_i$, is received only in state 1. For agents with $D_i < 0$, we will see that insurance is negative too, in the sense that the premium payment and indemnity benefits are both negative. If individual insurance is complete, we can be sure that $b_i = D_i$.

In addition to covering the cost of individual insurance, the state-contingent payments also pay for the public good. Let the state-independent “two-part premium” paid by $i$, which covers both her insurance premium and her share of the cost of the public good, be given by the continuous function $a_i : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}$, denoted $a_i(Z, D)$. Let $a(Z, D) = (a_1(Z, D), \ldots, a_n(Z, D))$. Where no ambiguity results, we will write $a_i$ and $a$. The planner then contracts for reinsurance in the aggregate amount $D$. The rate paid by the planner to the outside reinsurance market is determined by the exogenously given twice-differentiable pricing function $r : \mathbb{R}_+ \rightarrow (0, 1)$, written $r(Z)$, with $r' < 0$ and $r'' > 0$. The cost of the planner’s reinsurance contract with the market is $r(Z)D$.

**Definition 1.** The public project $(t, Z)$ is an insurance project, denoted $(a, Z)$, if for each $i$ and any $Z \geq 0$,

$$t^1_i = a_i(Z, D) - b_i \quad \text{and} \quad t^2_i = a_i(Z, D).$$  \hspace{1cm} (1)
We begin by assuming that fair reinsurance is available; this assumption is relaxed in Section 5. Reinsurance is fair if the outside market requires no compensation for the risk it assumes, in which case \( r(Z) = p(Z) \) for any \( Z \). A familiar textbook result, stated here for completeness, is that a risk-averse agent facing fair insurance will choose to insure fully.

**Proposition 1.** Suppose \( Z \geq 0 \). A risk-averse agent with endowment \( e_i \) who is offered insurance at the fair rate \( p(Z) \) will fully insure. Full insurance is strictly preferred to the endowed lottery.

Proposition 1 guarantees that, being fully insured, each agent will receive indemnity \( b_i = D_i \). Though the \( a_i \) functions are independent of the state, the transfer payments in (1) are not. For any value of \( Z \), at an insurance project each agent’s realized wealth is also independent of the state, with elements given by \( \gamma^j_i = w_i - (1 - p(Z))D_i \). This vector is \( E(e_i \mid p(Z)) \), and the vector of aggregate expected resources, given \( Z \), is \( \sum_i E(e_i \mid p(Z)) - c(Z) \).

Some means must be devised for distributing the cost of the public good across agents, within the framework of insurance payments. Our planner assumes that responsibility here, assigning a set of cost-sharing functions, resembling those in Serizawa (1999), that have the effect of distributing resources and risk among agents. Because the planner knows only damages, and not initial wealth or risk preferences, the shares are allowed to depend only upon the vector \( D \). For a given \( D \), then, define \( i \)'s cost share as the function \( s_i : \mathbb{R}^n \to \mathbb{R} \) given by \( s_i(D) \), possibly negative. A cost-sharing rule is \( s(D) = (s_1(D), \ldots, s_n(D)) \) with \( \sum_i s_i(D) = 1 \). The resulting contribution by \( i \) to the provision of the public good is \( s_i(D)c(Z) \). Given a cost-sharing rule, the two-part premium payment required of \( i \) is

\[
a_i(Z,D) = s_i(D)c(Z) + p(Z)D_i.
\]

The cost of insurance for \( i \) with \( D_i < 0 \) will be negative. When \( Z = 0 \), in which case \( c(Z) = 0 \), the payments in (2) reduce to insurance payments only.

**Definition 2.** An insurance project is a cost-sharing insurance project if the \( a_i(Z, D) \) are defined as in (2) for a given vector of cost shares with \( \sum_i s_i(D) = 1 \).

Henceforth, all of the insurance projects we consider will be cost-sharing insurance projects.

When reinsurance is available, resources are transferred in from outside in state 1 and transferred out in state 2. With complete insurance for each agent, all aggregate risk is removed and the level of resources available to the economy is equal in the two states.
Individual feasibility of an insurance project is based only upon information available to the planner. Say that \((a, Z)\) is individually feasible if (i) \(a_i \leq D_i\) for \(i\) with \(D_i > 0\), and (ii) \(a_i \leq 0\) for \(i\) with \(D_i < 0\). Feasibility requires individual feasibility and also affordability in each state at the probability \(p(Z)\).

**Definition 3.** The insurance project \((a, Z)\) with \(t_i^j\) defined as in (1) is feasible if it is (i.) individually feasible and (ii.) affordable in each state:

\[
\text{state 1: } c(Z) + p(Z)D \leq \sum_i t_i^1 + D, \quad \text{and}
\]

\[
\text{state 2: } c(Z) + p(Z)D \leq \sum_i t_i^2 .
\]

The inequalities in (3) and (4) are together equivalent to requiring that, at \(p(Z)\), the project is affordable in expectation: \(c(Z) + p(Z)D \leq \sum_i a_i(Z, D)\).

### 3 Efficient and individually rational insurance projects

We shall be interested in insurance projects that are both Pareto efficient and individually rational. Let \(\tilde{u}_i(a_i, Z; e_i)\) denote \(i\)'s expected utility when the two-part premium is \(a_i\) and the \(t_i^j\) satisfy equations (1)

**Definition 4.** The feasible insurance project \((a, Z)\) is Pareto efficient if there does not exist a feasible \((a', Z')\) such that, for each agent \(i\), \(\tilde{u}_i(a'_i, Z'; e_i) \geq \tilde{u}_i(a_i, Z; e_i)\), with the inequality strict for some agent \(j\).

A public project is individually rational (IR) if each agent prefers it to the endowed lottery \((e_i, \bar{p})\).

**Definition 5.** The public project \((t, Z)\) is individually rational (IR) if, for each agent \(i\), \(\tilde{u}_i(t_i, Z; e_i) \geq \tilde{u}_i(0, 0; e_i)\). The project is strictly individually rational if the inequality is strict.

Identifying the set of IR projects requires knowing each agent’s utility function, which our planner does not. Individual rationality of an insurance project has a different and distinctive nature. Say that an insurance project is insurance individually rational if each agent is no worse off than at the lottery \(E(e_i | \bar{p})\), with insurance at the premium rate \(\bar{p}\).

**Definition 6.** The insurance project \((a, Z)\) is insurance individually rational (IIR) if, for each agent \(i\), \(\tilde{u}_i(a_i, Z; e_i) \geq u_i(w_i - \bar{p}D_i)\). The project is strictly IIR if the inequality is strict.
Because complete insurance means that agents are indifferent as to which state occurs, the set of efficient insurance projects and the set of IIR insurance projects can both be computed without knowing agents’ utility functions. It is obvious that an IIR insurance project is also individually rational.

For any $Z > 0$, the condition on $a_i$ guaranteeing that a risk-averse agent will prefer the resulting insurance project $(a, Z)$ to the endowed lottery may be obtained using second-degree stochastic dominance. The following result provides the required condition. The proof relies upon the fact that, when comparing two lotteries, a risk-averse agent will strictly prefer a lottery that stochastically dominates in the second degree.

**Proposition 2.** Suppose that an insurance project $(a, Z)$ satisfies

$$a_i(Z, D) \leq \bar{p}D_i, \quad \text{all } i.$$  

Then $(a, Z)$ is IIR. If the inequality is strict for all $i$, then $(a, Z)$ is strictly IIR.

**Proof.** The argument is slightly different depending on the sign of $D_i$. If $D_i > 0$, the realization of wealth at the endowed lottery with $Z = 0$ is $w_i - D_i$ in state 1 and $w_i$ in state 2. At $Z > 0$, with complete insurance $i$’s certain wealth is $w_i - a_i(Z, D)$. The certain wealth dominates the endowed lottery by second-degree stochastic dominance if

$$[w_i - (w_i - a_i(Z, D))](1 - \bar{p}) \leq [(w_i - a_i(Z, D)) - (w_i - D_i)] \bar{p},$$

which may be rearranged to yield (5). If $D_i < 0$, with complete insurance $i$’s certain realization is again $w_i - a_i(Z, D)$, but $a_i$ must be negative. The certain realization dominates the endowed lottery by second-degree stochastic dominance if

$$[(w_i - D_i) - (w_i - a_i(Z, D))] \bar{p} \leq [(w_i - a_i(Z, D) - w_i)](1 - \bar{p}),$$

which may again be rearranged to yield (5).

Figure 1 illustrates. In both panels, individual rationality is assured so long as area $A$ exceeds area $B$.

Individual rationality may also be seen to depend explicitly upon the cost-sharing rule. Straightforward algebraic manipulation of (5) and (2) establishes that the corresponding IIR restriction on cost shares is that $s_i$ cannot exceed $i$’s insurance savings due to the public good, divided by its cost. Thus, consider an insurance project $(a, Z)$ with $Z > 0$. The
project is IIR if, for all $i$,

$$s_i(D; Z) \leq \left( \frac{\bar{p} - p(Z)}{c(Z)} \right) D_i.$$  \hspace{1cm} (6)

If the inequality is strict, the project is strictly IIR. If $Z = 0$, in which case $c(Z) = 0$ and so (6) is undefined, IIR is satisfied only if $a_i(Z, D) = \bar{p}D_i$ for all $i$. For a given $Z$, let $S^{irr}(Z) \subset \mathbb{R}^{n-1}$ denote the associated set of IIR cost shares. The associated set of strictly IIR cost shares is the interior of this set, denoted $\text{int}S^{irr}(Z)$. Note that for $i$ with $D_i < 0$, IIR requires that $s_i(D; Z)$ must be nonpositive. Because they are made worse off as the probability of their preferred state 1 declines, these agents will strictly prefer the project with $Z > 0$ only if they receive compensation by way of a negative cost share.

Figure 2 depicts the Edgeworth box for an insurance economy with two agents, each with CRRA utilities given by $u_i(c) = c^{1-\rho}/(1-\rho)$, and with $Z = 0$. Of the two, agent 2 is slightly more risk averse. The initial aggregate endowment is at $\sum e_i$ and the outline of the endowed box is given by the dashed lines meeting at that point. Because there is no public good, the relevant probability line has slope $\bar{p}/(1-\bar{p})$. Reinsurance removes aggregate risk and reshapes the Edgeworth box into a square. Its height, representing the resources available in state 2, is reduced relative to the endowment by $\bar{p}\sum_i D_i$, the aggregate cost of insurance. Its width is increased by $(1-\bar{p})\sum_i D_i$, the expected value of aggregate indemnity payments.

Individual endowments are at $e_1$ relative to $0_1$ and at $e_2$ relative to the northeast corner marked by $\sum E(e_i \mid \bar{p})$. Here we have that $D_1 > 0$ and $D_2 < 0$, so 1 would be made better off if $p$ were to decline (and the probability line to become flatter) and 2 would be made worse off. The dashed indifference curves pass through the initial endowment. The solid
Proposition 3 describes sufficient conditions for an insurance project to be Pareto efficient.

**Proposition 3.** Suppose that \( c'(0) < -p'(0) \sum_i D_i \). If a feasible insurance project \((a^*, Z^*)\),
with $t_i^1 = a_i^* - D_i$ and $t_i^2 = a_i^*$, satisfies

$$c'(Z^*) = -p'(Z^*) \sum_i D_i \tag{7}$$

then it is Pareto efficient. The efficient level of the public good is unique and strictly positive and the welfare surplus created by $Z^*$ is strictly positive.

**Proof.** We show first that the efficient level of the public good, $Z^*$, is unique. To see this, consider the problem of maximizing the financial surplus created by the public good. This is the state-independent quantity $(p(0) - p(Z)) \sum_i D_i - c(Z)$, the difference between the reduction in the cost of complete insurance and the cost of $Z$. The first-order necessary condition for this problem is equation (7). Given that $c'(0) < -p'(0) \sum_i D_i$, $c''(Z) > 0$, and $p''(Z) > 0$, the condition is also sufficient to guarantee that the solution $Z^*$ is unique.

The assumption that $c'(0) < -p'(0) \sum_i D_i$ also guarantees that there is $Z' > 0$ at which $(p(0) - p(Z')) \sum_i D_i > c(Z')$. Thus at the optimal $Z^*$ we must likewise have $(p(0) - p(Z^*)) \sum_i D_i > c(Z^*)$ and also $Z^*$.

With $a_i(Z, D)$ given by (2), expressions (3) and (4) are satisfied as equalities. Consider a vector of state-contingent payments $t^*$ that, together with $Z^*$, satisfies (1) and thus qualifies as an insurance project. By way of contradiction, suppose there exists $(t', Z^*)$ for which, for all $i$, $\bar{u}_i(t'_i, Z^*) \geq \bar{u}_i(t^*_i, Z^*)$ and, for some $j$, $\bar{u}_j(t'_j, Z^*) > \bar{u}_j(t^*_j, Z^*)$. Given that $u'_i(\cdot) > 0$ for all $i$, we know that, for $j$, either

$$t'^{1'}_j < a_j(Z^*, D) - D_j \quad \text{or} \quad \tag{8}$$

$$t'^{2'}_j < a_j(Z^*, D). \quad \tag{9}$$

Suppose (8) is true. Then

$$t'^{1'}_j + \sum_{i \neq j} t'^{1'}_i < a_j(Z, D) - D_j + \sum_{i \neq j} (a_i(Z, D) - D_i).$$

But then (3) is violated unless there is $k \neq j$ for whom $t'^{1'}_k > a_k(Z^*, D) - D_k$. This implies that $\bar{u}_k(t'_k, Z^*) < \bar{u}_k(t^*_k, Z^*)$, a contradiction. Now suppose (9) is true. Then

$$t'^{2'}_j + \sum_{i \neq j} t'^{2'}_i < a_j(Z, D) - D_j + \sum_{i \neq j} (a_i(Z, D) - D_i)$$

and so (4) is violated unless there is $k \neq j$ for whom $t'^{2'}_k > a_k(Z^*, D)$. Once again $\bar{u}_k(t'_k, Z^*) < \bar{u}_k(t^*_k, Z^*)$, another contradiction. This completes the proof. \qed
Note that the proof of Proposition 3 does not rely upon knowing the \( w_i \). Neither does Proposition 4, which establishes the nonemptiness of the set of Pareto-efficient and strictly IIR insurance projects.

**Proposition 4.** Consider an insurance economy for which \( c'(0) < -p'(0) \sum_i D_i \). The set of insurance projects \((a, Z)\) that are both Pareto efficient and strictly IIR is nonempty.

**Proof.** Take the optimal \( Z^* > 0 \) and note that the state-independent net surplus created by the public good is \( \Theta(Z^*) = (\bar{p} - p(Z^*)) \sum_i D_i - c(Z^*) > 0 \). Partition \( I \) into two subsets, the “winners,” denoted \( I^+ = \{i \in I \mid D_i > 0\} \), who gain when \( p \) falls, and the “losers,” denoted \( I^- = \{i \in I \mid D_i < 0\} \), who lose. Let \( n^+ = \#|I^+| \) and \( n^- = \#|I^-| \).

We derive a vector \( a(Z^*, D) \), giving nearly all of the surplus to the set of losers, for which the resulting \((a^*, Z^*)\) is PE and strictly IIR. Consider the set of winners \( I^+ \), for whom \( D_i > 0 \). Because \( D_i < 0 \) for \( i \notin I^+ \), we have that

\[
(\bar{p} - p(Z^*)) \sum_i D_i < (\bar{p} - p(Z^*)) \sum_{i \in I^+} D_i.
\]

The sum on the right is the aggregate surplus created for the set of winners by the public good, before imposing the cost of \( Z^* \). We know that \( c(Z^*) < (\bar{p} - p(Z^*)) \sum_i D_i \). Thus,

\[
c(Z^*) < (\bar{p} - p(Z^*)) \sum_{i \in I^+} D_i. \quad (10)
\]

Allocate the cost of \( Z^* \) among the members of \( I^+ \) proportionally, so that \( i \) contributes

\[
\left( \frac{D_i}{\sum_{i \in I^+} D_i} \right) c(Z^*) < (\bar{p} - p(Z^*)) D_i,
\]

where the inequality can be seen to hold by summing over \( i \in I^+ \) to obtain (10). Define

\[
a^*_i(Z^*, D) = (\bar{p} - p(Z^*)) D_i - \left( \frac{D_i}{\sum_{i \in I^+} D_i} \right) c(Z^*) - \varepsilon. \quad (11)
\]

For any \( \varepsilon > 0 \), this \( a^*_i \) is strictly IIR for all \( i \in I^+ \).

The surplus remaining after the winners have paid the two-part premiums in (11) is the sum of those payments,

\[
\sum_{i \in I^+} a^*_i(Z^*, D) = (\bar{p} - p(Z^*)) \sum_{i \in I^+} D_i - c(Z^*) - n^+ \varepsilon.
\]
For \( \varepsilon \) sufficiently small, this is greater than \((\bar{p} - p(Z^*)) \sum_{i \in I^-} D_i\), the aggregate losses created for the losers by \( Z^* \). Choose \( \varepsilon \) this small and allocate to each \( i \in I^- \) the state-independent premium

\[
a^*_i(Z^*, D) = (\bar{p} - p(Z^*)) D_i - \left( \frac{D_i}{\sum_{j \in I^-} D_j} \right) (\Theta - n^+\varepsilon).
\]

(12)

The first term on the right in (12) is the (negative) payment required to make \( i \in I^- \) indifferent between \((a^*, Z^*)\) and insurance at \( \bar{p} \). The second term sweetens the pot and causes \( i \) to prefer \((a^*, Z^*)\) strictly. One may easily check to see that the sum of the \( a^*_i \), using (11) for \( i \in I^+ \) and (12) for \( i \in I^- \), equals \( c(Z^*) + p(Z^*) \sum_i D_i \). Thus, the budget balances in both states and the project is both PE and strictly IIR.

From (11), by setting \( \varepsilon = 0 \) one may obtain the largest cost share that can be extracted from a winner while guaranteeing that the resulting two-part premium is just IIR. Insert the expression for \( a^*_i(Z^*, D) \) from (11) into (6) and rearrange to obtain

\[
s_i(D; Z) = \frac{D_i}{c(Z^*)} \left[ \frac{(\bar{p} - 2p(Z^*)) \sum_{j \in I^+} D_j - c(Z^*)}{\sum_{j \in I^+} D_j} \right].
\]

The corresponding expression for a loser, negative in this case, may be obtained from (12).

Figure 3 depicts the situation that results when the optimal \( Z^* \) is added to the example from Figure 2. The two panels are not Edgeworth boxes, but rather show both agents’ preferences relative to the same origin. The dashed indifference curves in panel (a) correspond to the \( \gamma_1^I \), where insurance is available at the endowed \( \bar{p} \). The solid indifference curves correspond to the optimal probability with \( Z^* \), represented by the lines labeled \( p(Z^*) \), one through each agent’s endowment. Agent 1 is made better off (moving from \( \gamma_1^I \) to \( \gamma_1(p^*) \)) as a result of the public good, but this comparison is before accounting for (i) the cost of \( Z^* \), and (ii) the payment that must be made to agent 2. Before this compensating payment is received, agent 2 is worse off at \( \gamma_2(p^*) \) than at the initial insurance outcome \( \gamma_1^I \).

The right and upper boundaries of the square in panel (b) describe a smaller square than that in (a), by the cost of \( Z^* \). A surplus due to the public good remains. The point labeled \((e_1 - a^1_{\max})\) is 1’s state-contingent outcome after paying the largest premium payment she can pay without being worse off than at the insurance outcome \( \gamma_1^I \) at \( \bar{p} \). The point labeled \((e_1 - a^1_{\max})\) reflects the outcome received by 1 when 2 captures the entire surplus and pays \( a^2_{\min} < 0 \). Agent 1 is now indifferent between the project and \( \gamma_1^I \) at \( \bar{p} \). The two bolded segments, one southwest of \( e_1 \) along a line of slope 1 and one northeast of \( e_2 \) along a line of the same slope, together trace out the set of feasible and IIR two-part premium payments.
Figure 3: Cost sharing for the optimal public good, $Z^* > 0$. (a) Public good and $p(Z^*)$. (b) IIR cost shares accounting for $c(Z^*)$. 
between agents that leave both at least as well off as at their respective $\gamma_i^t$, where $Z = 0$ and $p = \bar{p}$.

4 Implementing PE and IIR insurance projects

The foundation has now been laid for a description of the mechanism that implements the set of PE and IIR insurance projects. Together with the external reinsurance market, an insurance economy consists of (i) the set $I$ of agents and their endowments and utility functions; (ii) the functions $p(Z)$ and $c(Z)$; and (iii) the planner. The set of insurance economies $E$, with generic element $\mathcal{E}$, are those in which these elements satisfy the conditions placed upon them (risk-averse utility, derivative restrictions on $p(Z)$ and $c(Z)$, $\gamma^t_i$ is given by (1), and so on). The premium vector $a$ is permissible given $Z$ if there is a share vector $s$ for which the $a_i(Z, D)$ satisfy (2) at $Z$ and $(a, Z)$ is feasible. Let $A(Z)$ be the set of permissible two-part premium vectors given $Z$.

Given the set of insurance economies $E$, a choice correspondence $F : E \Rightarrow A \times \mathbb{R}^+ \times \mathbb{R}^+$ is a mapping from an economy to a set of insurance projects $(a, Z)$. We shall be interested in choice correspondences whose values are both Pareto efficient and insurance individually rational. The Pareto-efficient and strictly IIR choice correspondence at $\mathcal{E}$ is $\Pi(\mathcal{E}) = \{(a, Z) \mid (a, Z) \text{ is both PE and strictly IIR}\}$.

A mechanism is a game form $G$, consisting of a strategy space together with an outcome function $h$. Players include agents and the planner, who is given the index $i = n + 1$. The strategy set of agent $i = 1,\ldots,n$ is $M_i = \{0, 1\}$, with element $m_i$. The planner’s strategy set is defined as $M_{n+1} = \mathbb{R}_+ \times \mathbb{R}^{n-1}$ with element $m_{n+1} = (\zeta, s)$, where $\zeta$ represents the planner’s proposed level of $Z$ and $s$ represents a vector of cost shares satisfying $\sum_i s_i = 1$. The strategy space is $M = M_1 \times \cdots \times M_n \times M_{n+1}$, with element $m$. An outcome function is $h : M \rightarrow \mathbb{R}^{2n} \times \mathbb{R}_+$, mapping the strategy space into vectors of allocations and the public good, $(\gamma, Z)$. Let $h_Z(m)$ and $h_i(m)$ denote the level of $Z$ and of $i$’s vector of realized wealth, respectively, resulting from strategy vector $m$. Combining $G$ with an economy $\mathcal{E}$ defines the game $(G, \mathcal{E})$.

Given an outcome function, preferences on strategies are defined as follows. Let indirect expected utility for $i = 1,\ldots,n$ be given by $v_i(m) = \tilde{u}_i(h_i(m), h_Z(m))$. For $i = 1,\ldots,n$, the strategy vector $m$ is preferred to $m'$ by $i$ if $v_i(m) \geq v_i(m')$, with strict preference when the inequality is strict. The payoff function for $i = 1,\ldots,n$ is simply $v_i(m)$. The planner’s
payoff function is the monetary surplus created by the public good:

\[ v_{n+1}(\mathbf{m}) = (\bar{p} - p(h_Z(\mathbf{m}))) \sum_i D_i - c(h_Z(\mathbf{m})). \]  

(13)

Note that at any \( Z \) the planner is indifferent regarding the allocation.

The equilibrium concept employed in our mechanism is that of equilibrium in undominated strategies. A strategy \( m_i \) is undominated for \( i \) if, for any \( m_{-i} \) played by the other players, no strategy \( m'_i \) yields a higher payoff than \( m_i \).

**Definition 7.** A strategy \( m_i \) is **dominated** for \( i \) if there exists \( m'_i \) such that for all \( m_{-i} \),

\[ v_i(m'_i, m_{-i}) \geq v_i(m_i, m_{-i}), \]

with strict inequality for some \( m_{-i} \). The strategy \( m_i \) is **undominated** if there does not exist such an \( m'_i \).

**Definition 8.** A strategy vector \( \mathbf{m}^* \in \mathbf{M} \) is an **equilibrium** in undominated strategies (henceforth, an “equilibrium”) if for all \( i = 1, \ldots, n + 1 \), \( m_i^* \) is undominated.

The correspondence \( d(G, \mathcal{E}) = \{ h(\mathbf{m}) \mid \mathbf{m} \text{ is an equilibrium in undominated strategies in } \mathbf{M} \} \) associates with each economy \( \mathcal{E} \) the set of equilibrium outcomes. A game form with strategy space \( \mathbf{M} \) and outcome function \( h \) **fully implements** \( \text{PI}(\mathcal{E}) \) in undominated strategies if, for any \( \mathcal{E} \), \( \text{PI}(\mathcal{E}) = d(G, \mathcal{E}) \).

A key to the mechanism is that, as shown in the proof of Proposition 3, at an efficient project the public good reduces the cost of insurance by an amount greater than the cost of the public good itself. This surplus may be used to devise a combination of insurance and \( Z \) that, so long as the cost-sharing rule is chosen judiciously, will be advantageous to all agents without respect to risk preferences and regardless of others’ strategies. Another key is that at the offered terms, each agent will purchase complete insurance, which renders risk preferences irrelevant.

The mechanism consists of two stages. In the first stage, the planner selects \( m_{n+1} = (\zeta, s) \). In the second stage, each agent responds with \( m_i = 0 \) or 1. The outcome function consists of two components. The level of the public good is \( \zeta \) if all agents choose \( m_i = 1 \) and zero otherwise:

\[ h_Z(\mathbf{m}) = \begin{cases} 
\zeta & \text{if } \sum_i m_i = n \\
0 & \text{otherwise.}
\end{cases} \]

(15)
The allocation is determined according to
\[
    h_i(m) = \begin{cases} 
        (w_i, w_i - D_i) & \text{if } m_i = 0 \\
        (w_i - \bar{p}D_i, w_i - \bar{p}D_i) & \text{if } m_i = 1 \text{ and } \sum_j m_j < n \\
        (w_i - s_iC(\zeta) - p(\zeta)D_i, w_i - s_iC(\zeta) - p(\zeta)D_i) & \text{if } \sum_j m_j = n. 
    \end{cases} 
\]
(16)

That is, each agent is offered a combination of the public good \(\zeta\) and insurance at price \(p(\zeta)\), together with two-part premium \(a_i(\zeta, D)\) as determined by the planner's announced cost shares, \(s\). If all agents accept the offer \((m_i = 1)\), the public good is provided at the level \(\zeta\) and the insurance portion of the package is priced at \(p(\zeta)\). If one or more agents decline the offer \((m_i = 0)\), \(Z\) is set to zero. Those who decline the offer are left with their endowed lottery, paying nothing and receiving neither insurance nor the public good. Those who accept the offer receive insurance at the rate \(\bar{p}\).

We are now prepared to state and prove our main result.

**Proposition 5.** Consider an insurance economy \(E \in E\). The game form \(G\) defined by the strategy space \(M\) and the outcome function in (15) and (16) fully implements the Pareto-efficient and insurance individually rational choice correspondence in undominated strategies.

**Proof.** The first step is to show that any efficient and IIR insurance project \((a^*, Z^*)\) must be an equilibrium outcome in the associated game. Take such an outcome, and note that to \(a^*\) may be assigned a unique vector of cost shares, which we denote \(s(a^*)\). It is clearly the case that \(\zeta = Z^*\) is undominated for the planner.

We claim that \(m_i = 1\) is also undominated for player \(i\). To see this, suppose that at least one agent \(j\) plays \(m_j = 0\), in which case the outcome of the game will include \(Z = 0\). From Proposition 1 agent \(i\) strictly prefers full insurance at rate \(\bar{p}\) to the endowed lottery. Thus, we conclude immediately that \(m_i = 1\) is undominated if \(\sum_{j \neq i} m_j < n - 1\). Now suppose that \(\sum_{j \neq i} m_j = n - 1\), so that \(i\) is pivotal in the sense that playing \(m_i = 1\) causes the public good to be built. We must show that for \(Z^*\) and a cost share \(s \in S^{IIR}(Z^*)\),
\[
    \tilde{u}_i((\bar{p}D_i - D_i, \bar{p}D_i, 0; w_i, D_i) \leq \tilde{u}_i(a_i(Z^*, D) - D_i, a_i(Z^*, D), Z^*). 
\]
(17)

Given \(s \in S^{IIR}(Z^*)\), we must have \(|a_i(Z^*, D)| \leq |\bar{p}D_i|\). By monotonicity of the \(u_i\), then, we know that
\[
    u_i(w_i - \bar{p}D_i) \leq u_i(w_i - a_i(Z^*, D)). 
\]
(18)
But (18) can be seen, upon expanding and rearranging, to be equivalent to
\[
\bar{p}u_i\left(w_i - \bar{p}D_i - D_i + D_i\right) + (1 - \bar{p})u_i\left(w_i - \bar{p}D_i\right) \leq p(Z^*)u_i\left(w_i - a_i(Z^*, D) - D_i + D_i\right) + (1 - p(Z^*))u_i\left(w_i - a_i(Z^*, D)\right),
\]
which in turn is simply (17) with the expected-utility terms expanded. We conclude that the strategy \(m_i = 1\) is undominated for all \(i\) at \(Z^* > 0\) and \(s \in S_{IIR}(Z^*)\). Because \(a^*\) was taken to be an arbitrary member of the set of IIR premium vectors, every element of \(PI(E)\) is an equilibrium outcome.

The second step is to show that any equilibrium outcome in the game must be an efficient and IIR insurance project in the associated insurance economy. Take an equilibrium vector \(m^*\) and the resulting outcome \(h(m^*)\). At equilibrium we know that \(\zeta = Z^*\) because any \(Z \neq Z^*\) is dominated for the planner by \(Z^*\) in the event that \(m_i = 1\) for \(i = 1, \ldots, n\). It must be shown that, together with \(Z^*\), the equilibrium outcome \(a^*\) must be both PE and IIR in the economy. At an equilibrium it must be the case that \(m_i = 1\) for \(i = 1, \ldots, n\) because player \(i\) is strictly better off playing \(m_i = 1\) when \(Z^*\) is chosen and some \(j \neq i\) plays \(m_j = 0\). But if each agent plays 1, given that insurance is fair, the resulting project must be Pareto efficient. Given that the planner’s equilibrium choice \(s \in S_{IIR}(Z^*)\), the resulting project is also insurance individually rational.

It is straightforward to show that, if the planner is restricted to choosing an \(s\) in the interior of \(S_{IIR}\), the strategy \(m_i = 1\) is strictly dominant for \(i = 1, \ldots, n\).

The planner has a certain amount of discretion regarding the distributional effects of the cost shares. For example, the planner might, perhaps for political reasons, choose to offer a zero share to each agent with \(D_i < 0\), spreading the cost across those who benefit from the public good. Note that proportional shares, with \(s_i(D, Z) = D_i/\sum_i D_i\), do not lead to an IIR insurance project.

The ability to pursue distributional concerns is limited by the fact that the planner does not know agents’ endowed wealth. The richest agent in the economy, having the largest \(w_i\), might have a small or negative \(D_i\). A planner who had complete information regarding both wealth and damages could choose to place the greatest cost burden on those with the greatest expected wealth, for example. Or she could choose shares according to a Rawlsian criterion that protects the poor by imposing the entire cost on those with the greatest initial wealth. We leave for future work a more detailed investigation of such equity considerations.
5 Costly reinsurance

In practice, of course, even a competitive exogenous reinsurance market may not be willing to provide coverage at a fair rate. In this section we explore briefly how our results change if the price of insurance exceeds the probability of loss: \( r(Z) > p(Z) \). In this case a load, given by

\[
L(Z, D) = (r(Z) - p(Z)) \sum_i D_i,
\]

must be paid to the reinsurer. Recall the expression in equation (13) for the economic surplus created by the public good with insurance priced at \( p(Z) \). The mechanism described above can be modified to to perform successfully so long as, at the optimal \( Z^* \), the load is not greater than the surplus:

\[
L(Z^*, D) \leq (\bar{p} - p(Z^*)) \sum_i D_i - c(Z^*), \quad \text{or} \quad 0 \leq (\bar{p} - r(Z^*)) \sum_i D_i - c(Z^*). \tag{19}
\]

The right side of (19) is the economic surplus remaining after the payment of reinsurance at a rate of \( r(Z^*) \).

If (19) is satisfied, a slightly modified version of our mechanism can still fully implement the PE and IIR projects. The modification is necessary because of a new difficulty that arises if one agent declines the project on offer. Now there is no surplus out of which to pay the load, and so no other agent can be offered fair insurance either. Players and strategy sets remain unchanged, as does the outcome function for \( Z \) given by (15). All that changes is the outcome function for the allocation, \( a \). Replace (16) with

\[
h_i(m) = \begin{cases} (w_i, w_i - D_i) & \text{if } \sum_j m_j < n \\ (w_i - s_i C(\zeta) - p(\zeta) D_i, w_i - s_i C(\zeta) - p(\zeta) D_i) & \text{if } \sum_j m_j = n. \end{cases} \tag{20}
\]

At an equilibrium of this mechanism the planner selects the optimal \( Z^* \) to maximize the economic surplus (using \( p(Z) \), not the pricing function \( r(Z) \)) created by the public good. Agents are again offered fair insurance, priced at \( p(Z) \). After the insurance payments of \( p(Z^*) \sum_i D_i \) are collected, the remaining cost of the project is now the sum of the load and the cost of the public good:

\[
L(Z^*, D) + c(Z^*) = (r(Z^*) - p(Z^*)) \sum_i D_i + c(Z^*).
\]
The planner selects a vector of cost shares and computes two-part premiums according to

\[ a_i(Z^*, D) = s_i(D; Z^*) \left[ (r(Z^*) - p(Z^*)) \sum_i D_i + c(Z^*) \right] + p(Z^*) D_i. \]

Given that we have assumed (19) is satisfied, an argument along the lines of that found in Proposition 4 ensures that the set of PE and IIR allocations is nonempty. Thus, cost shares can be selected so that each agent prefers the allocation under the insurance project, with insurance priced at \( p(Z^*) \), to the endowed lottery. All agents will insure fully, and so their risk preference remain irrelevant. A slight variant of the proof of Proposition 5 goes through. The substantative difference is that now, by choosing \( m_i = 0 \), a single defector leaves all other agents with nothing, not even insurance at the rate \( \bar{p} \). None will defect.

If condition (19) is not satisfied, then the insurance project is simply infeasible. One may easily imagine such cases, and determining which local risks can be insured using exogenous reinsurance is an empirical matter. The local economy may still improve its situation using Graham’s approach to risk sharing and the provision of public goods. That approach, as we have said, is not practicable unless the planner knows agents’ utility functions.

6 Conclusions

One typically imagines that the introduction of uncertainty increases complexity and therefore also increases the difficulty of achieving desirable outcomes. Thus the condition describing an uncertain Pareto-efficient outcome in Graham is more subtle and more complicated than its counterpart in Samuelson.

For the problem examined in this paper, quite the opposite is true. We are able to achieve implementation in undominated strategies of the set of Pareto-efficient and individually rational insurance projects. It might be said that we claim too much, that the comparison does not quite ring true. After all, when complete insurance is available, even risk-averse agents become indifferent over which state occurs. Thus we are not attempting to tease out information about risk preferences. Rather, under our insurance scheme risk preferences are rendered moot.

Does the introduction of a reinsurance market make our results less compelling than otherwise? We believe not. While it is true that Graham does not resort to such a device, it seems to us that the better argument lies in our favor. Reinsurance, even when it is not actuarially fair, confers a real economic benefit upon our economy. And it more closely resembles actual insurance markets, where reinsurance is an essential component. Another
attractive feature of our approach is that, unlike most straight taxes, our insurance scheme creates no deadweight loss. Quite the opposite: it enhances social welfare instead, though we are quick to point out that the same is true of Graham’s state-dependent individual transfers.

Another possible complaint is that our assumption that losses, our \( D_i \), are observable (and verifiable) is too strong. In a strict sense this might be true, yet in actual insurance markets we find that contracts depend upon insurers’ ability to estimate insurable losses quite closely. A great deal of effort is put into such estimates, and it is perhaps reasonable to suppose that insurance companies who are not adept in this area do not long remain in the industry.

The fundamental idea animating the paper is of real practical importance. The World Bank (2013) recently launched a pilot program to provide reinsurance to a group of five Pacific Island Countries against hurricane, tsunami, and earthquake risk. The World Bank plays something like the role of our planner, serving as an intermediary between the countries and the international reinsurance market. An important difference is that the plan does not support the construction of protective public goods. Countries’ payments also insure one another, which the Bank estimates leads to a savings of some 65% in the cost of reinsurance relative to the cost if each country obtained insurance alone. If one views each country as an agent, the linkage between our model and the pilot program is striking. An important difference is that while we have only two states (either everyone suffers loss or no one does), each island country can be struck by a natural disaster. The necessary extension to the model presented here is left for future work.
References


