Real Options and Environmental Economics: An Overview

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I. Real Options In A Nutshell

- Example: risk neutral planner

\[
\begin{align*}
\text{Time:} & \quad \text{now} & \quad \text{next year} & \quad \text{forever} \\
\text{Payoff:} & \quad 10 & \quad 5 & \quad 5 \\
\text{discount rate} & = 10\% \\
\text{cost of investment} & = 84
\end{align*}
\]

- Expected NPV = $10/0.1 - 84 = $16
  - Go ahead: invest now
Example cont’d

BUT: what if waiting till next year to decide?

- If unfavorable ($5), 5/.1 < 84:
  - Don’t invest!
- If favorable ($15), 15/.1-84= $66
  - Invest
- Expected payoff = (.5)(66)/1.1=$30
- Should *not* invest now! (30 > 16)
- Delay helps avoid unfavorable investment that you will regret given the new information
What is the story?

- **Hysteresis**: waiting has value when
  - There is **uncertainty** in payoff of investment
  - You can **learn** in the future by delaying
  - You can **delay** the investment
  - Investment is **irreversible** or costly reversible

- **The value is called** **option value**
  - Much like financial option value
    - Example: call option: opportunity to invest in year two
    - Value is $30

  - Investment now kills this option

  - Invest now only if ENPV ≥ OV, or if the benefit can cover both the cost and the OV

- **Investment now competes not only with no-investment, but also with investment later**
II. A Brief History

- Weisbrod (1964)’s conjecture
  - Park has value even if I don’t visit it
  - Reason: possible visits, in the future

- Two interpretations of Weisbrod
  - Option price, due to risk attitude
    - Zeckhauser (69), Cicchetti and Freeman (71), Ready (’95)
    - Risk premium (or option value): difference between WTP and expected CS, or \textit{ex ante} and expected \textit{ex post} welfare measures
    - No dynamic decision
    - But, can be negative, depending on the concavity/convexity of marginal utility functions
  - \textbf{(Quasi-)} option value: due to arrival of new information
    - Maintain the flexibility of responding to new information
    - Independent of risk attitude
    - Dynamic framework with learning
    - Always positive
    - Conditional value of information
The OV literature

- Branching Out:
  - Information service, Bayesian updating
    - Epstein (’80), Freixas and Laffont (’84), Jones and Ostroy (’84), Demers (’91)
  - Role of information, ranking of informativeness (Blackwell’s measure)
  - Mostly discrete time, two or three periods
- The Dixit-Pindyck framework
  - Much like financial modeling, similar to Black and Scholes
  - Information follows a stochastic process
  - New info: new observed value of the variable

Applications
- Res., env., and ag., economics
- General econ: labor, investment, exchange rate, real estate
- Industrial engineering: capital budgeting, to account for managerial flexibility
III. The Dixit-Pindyck Framework

- Basic Idea: McDonald and Siegel (1986)
  - An investment project whose value $V_t$ follows geometric Brownian motion:
    $$dV_t = \alpha V_t dt + \sigma V_t dz_t$$
  - $dz_t$ is increment of Weiner process
    - $dz_t \sim N(0, dt)$: “scale” of $dz_t$ is $\rho dt$
    - $dz_t$ and $dz_s$ are independent, for $t \neq s$
    - Typical of stock prices
  - Decision problem:
    - When to incur cost of $I$ to lock in the project
    - Or at what value of $V_t$ to invest
    - If $V_0 = V$, and discount rate is $\rho$ (maybe risk adjusted), then ($\alpha < \rho$)
    $$F(V) = \max_T E e^{-\rho T} [V_T - I]$$
Two Solution Methods:

- **Contingent claims analysis**
  - Similar to valuation of financial options: another version of Black and Scholes
  - Applicable when the risk $dz_t$ can be spanned by existing assets in financial markets: *rich* set of assets
  - Market has to be in equilibrium: no arbitrage
  - Can value $F$ without any assumption about the discount rate or the investor’s risk attitude (without knowing $\rho$):
    - The price of the *option* is *relative to* other assets that are traded in the market

- **Dynamic programming**, or optimal stopping
  - Has to assume a discount rate
  - Applicable to many environmental problems
III.1 Solution method: DP

Bellman equation for $F(V(t))$

$$F(V(t)) = \max\{V(t) - I, e^{-\rho dt} E[F(V(t + dt))]\}$$

- Not straightforward to solve: discrete decision
- Trick: transform into optimal stopping
  - Exists a critical value $V^*$ so that
    - Continuation region: wait if $V < V^*$
      $$F(V(t)) = e^{-\rho dt} E[F(V(t + dt))]$$
    - Stopping region: invest if $V \geq V^*$
      $$\Omega(V(t)) = V(t) - I$$
  - At $V^*$ (due to $\max\{\varphi, \emptyset\}$)
    Value matching: $F(V^*) = \Omega(V^*)$
    Smooth Pasting: $F(V^*) = \Omega_V(V^*)$
Optimal stopping

- Conditions for connected regions, divided by $V^*$
  - Monotonicity conditions for both payoffs and distribution of $V(t+dt)$ given $V(t)$
  - Satisfied by most problems
  - Intuition: if $V$ is high, the opportunity cost of waiting, $V-I$, is high

- Value matching and smooth pasting conditions
  - VMC: intuitive, true if both $F(\xi)$ and $\Omega(\xi)$ are continuous
  - SPC: trickier, true if both functions are continuously differentiable (Dixit 1993)
Optimal stopping, with VMC and SPC

\[ \Omega(V) \]

\[ F(V) \]

continuation region

stop region
The continuation region

\[ F(V(t)) = e^{-\rho dt} E[F(V(t + dt))] \]

Rewrite the equation

\[ F(V(t)) = (1 - \rho dt)(F(V(t)) + EdF(V(t))) \]

Letting \( dt \to 0 \)

\[ \rho F(V) = \frac{E(dF(V))}{dt} \quad \text{Expected return} = \rho \]

Apply Ito’s Lemma

\[ dV_t = \alpha V_t dt + \sigma V_t dz_t \]

\[ dF(V) = F'(V) dV + \frac{1}{2} F''(V) (dV)^2 \]

\[ = F'(\alpha V dt + \sigma V dz) + \frac{1}{2} F''(\sigma^2 V^2 dt + o(dt)) \]
Ordinary differential equation

\[ \frac{1}{2}\sigma^2 V^2 F''(V) + \alpha V F'(V) - \rho F(V) = 0 \]

Boundary conditions are provided by VMC and SPC, as well as the natural economic condition (free boundary!)

\[ F(0) = 0 \]
\[ F(V^*) = V^* - I \]
\[ F'(V^*) = 1 \]

Guess a solution to the PDF: \( F(V) = AV^\beta \)

Fundamental quadratic:

\[ \frac{1}{2}\sigma^2 \beta (\beta - 1) + \alpha \beta - \rho = 0 \]

Roots: \( \beta_1 > 1 \), decreasing in \( \sigma \);
\( \beta_2 < 0 \), increasing in \( \sigma \)
Solution

General solution:

\[ F(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2} \]

Impose the boundary conditions

\[ F(0) = 0 \]
\[ F(V^*) = V^* - I \]
\[ F'(V^*) = 1 \]

\[ V^* = \frac{\beta_1}{\beta_1 - 1} I \]
\[ A_1 = \text{some constant} \]
Interpretation of the results

- **Hysteresis: \( V^* > I \)**
  - More reluctant to invest, compared with neoclassical investment rule (\( V^* = I \))
    - Don’t want to jump as \( V \) may rise further
    - VMC \( V^* = I + F(V^*) \): return from investment has to overcome both cost \( I \) and option value \( F \)
  - Investment barrier increases
    - As uncertainty rises: \( V^* \) increasing in \( \sigma^2 \)
    - As \( \rho \) decreases: cost of waiting goes down

- **Investment barrier vs. probability of investment**
  - Move in same direction if exogenous changes do not affect the distribution of \( V_t \)
  - As \( \sigma^2 \) rises, investment prob may rise or fall (Sarkar, 2000)

\[
V^* = \frac{\beta_1}{\beta_1 - 1} I
\]
III.2 Solution method: contingent claims

Optimal stopping by definition:

Holding an option $F(V)$, and when to exercise it?

Suppose there exist spanning assets, replicating the risk $dz$

$$dx = \mu x dt + \sigma x dz$$

Market equilibrium:

CAPM: $\mu$ is determined by the market

$$\mu = r + \phi \rho_{xm} \sigma$$

Exercising the option

Assume $\mu > \alpha$, otherwise, will never exercise the option

Convenience yield, or dividend rate: $\delta \, \mu - \alpha$
Forming a riskless portfolio

- Long one option: \( F(V) \)
- Short \( n = F'(V) \) units of \( x \), or the investment project
- Value of the portfolio: \( \Phi = F - F'(V) V \)
- Return from the portfolio over \( dt \)
  - Change in value (capital appreciation): \( dF - ndV \)
  - Dividend payout: \( \delta V n \ dt \)
  - Total return: \( dF - F'(V) dV - \delta V F'(V) \ dt \)
  - Applying Ito's Lemma to \( dF \)
    \( dF = F'(V) dV + .5 F''(V) \sigma^2 V^2 \ dt \)
  - Deterministic total return:
    \( (1/2)\sigma^2 V^2 F'' \ dt - \delta V F' \ dt \)
- Equilibrium: return = \( r \)
  \( (1/2)\sigma^2 V^2 F'' \ dt - \delta V F' \ dt = r \Phi \ dt = r(F-F'V)dt \)
- Similar ODE:
  \[ \frac{1}{2} \sigma^2 V^2 F''(V) + (r - \delta)VF'(V) - rF(V) = 0 \]
Compare with DP

- The same boundary conditions: VMC and SPC
- Compare the ODEs

\[ \frac{1}{2} \sigma^2 V^2 F''(V) + \alpha VF'(V) - \rho F(V) = 0; \quad \text{(DP)} \]

\[ \frac{1}{2} \sigma^2 V^2 F''(V) + (r - \delta)VF'(V) - r F(V) = 0; \quad \text{(Mkt equil)} \]

Risk neutral valuation:
- Replace \( \rho \) by \( r \)
- Replace expected return \( \alpha \) by \( (r - \delta) \), valued under the risk neutral probability
III.3 Extensions of the basic model

- **Endogenous process of dV**
  - Production with variable output, temporary suspension, price uncertainty
  - Solution: find process for V first
  - Essentially the same results

- **Different stochastic processes**
  - Mean-reversion
  - Poisson jump
  - Reflecting barriers

- **Entry and exit (invest and disinvest)**
  - Sunk fixed fees for entry and exit
  - Reluctant to do either
    - Entry: future price may go down (regret!)
    - Exit: future price may go up (regret!)
    - Area of inaction
Entry and exit: two barriers

Invest Region

No-action Region

Disinvest Region
III.3 Extensions (cont’d)

- **Continuous investment levels**
  - Choose how much to invest, rather than whether invest or not
  - Trick: decide the marginal unit, or the last unit
    - If willing to invest this unit, all earlier units should be invested
    - Similar results

- **Multiple stages**
  - A project may require many stages to complete
  - Each stage incurs sunk cost
  - Most reluctant to start earlier stages:
    - More info at later stages
    - Higher loss if regret
Extensions

- Competitive equilibrium
  - No monopoly in investment opportunity
  - If wait, other firms may invest, driving down the price
  - Surprise: the same investment rule (Leahy, 1993; Baldursson and Karatzas, ’97; Zhao, forthcoming)
  - Intuition:
    - Entry of other firms: price ceiling
    - Investment today competes with investment tomorrow
    - Price ceiling reduces both values, without changing their relative value
Recent Extensions

- Double sided irreversibility
  - Kolstad, JPubE, 1996
  - Both abatement investment and global warming damages are irreversible
  - Investment depends on the relative prob and costs of the two irreversibilities

- Multiple options
  - Some research in capital budgeting, Trigeorgis, 1993
  - Depends on whether the multiple stages are complements and substitutes (Weninger and Zhao, 2002)
    - Willing to invest early if complements: creates more future flexibility
    - Less willing to invest if substitutes, in order to preserve future flexibility
Recent extensions

- **Strategic interactions**
  - Not much research: Dutta and Rustichini, ET, 93
  - The strategic relationship may increase or decrease the value of remaining flexible, depending on the form of interaction

- **Endogenous learning**
  - Miller and Lad, 1984
  - Experimentation literature (Mirman et al, 92, 93,..)

- **Empirical research**
  - Econometrics
    - Very few: Paddock, et al. QJE, 1988; Quigg, 1993;
  - Simulation: growing (Slade, 2001)
  - Structural estimation (Rust’s methodology)?
IV. Applications in Env. & Res. Econ.

- **General applications**
  - **Resource extraction, development and management** (Brennan and Schwartz, ’85a,b; Stensland and Tjostheim, ’85; Paddock, Siegel and Smith, ’88; Trigeorgis, ’90; Lund, ’92; Rubio, 1992; Zhao and Zilberman, ’99; Mason, ’01; Weninger and Just, 2002)
  - **Species preservation** (Krutilla, 64; Fisher, Krutilla and Cicchetti, ’72; Fisher and Hanemann, 1986)
  - **Global warming** (Nordhaus, ’91; Ulph and Ulph, ’97; Kolstad, ’96a,b)
  - **Abatement investment under different policies** (Xepapadeas, ’99; Chao and Wilson, ’93; Zhao, forthcoming)
Applications

- Policy making, endogenous irreversibility
  - Pindyck, 2000: a new policy may be hard to reverse
    - Gradual changes in policy, rather than one big decision
  - Zhao and Kling, 2002:
    - Initial policy change may set a trend that is hard to reverse
    - Then even more cautious
      - Similar to facing a fixed cost
      - Very reluctant to change initially, but once decides to change the policy, the change is relatively big
Environmental policy

Figure 3: Optimal Policies Before and After the Policy Trend is Set
Application: env. valuation, WTP/WTA

- Key result in applied welfare analysis:
  - CV = WTP and EV=WTA (for price decrease, quality increase)
  - WTP ¼ WTA, except for income effects (and later on, Hanemann’s substitution effects)
  - Behavior based measurements vs. value measurement

- A typical CVM study:
  - How much are you willing to pay to preserve a park
  - WTA to get rid of it
  - WTP/WTA values are taken as measures of CV/EV
However,

- If the subject
  - Is uncertain about the value of the park or substitutes/complements
  - Expects that she can learn about the value
  - Has some willingness to wait
  - Expects a cost of reversing the action of buying or selling (the only survey!)

- Then, she may choose to wait for more info before making a decision

- But, in surveys/experiments, she has to form a WTP or WTA offer now, with existing info
  - She needs compensation for the lost option value
  - Lower WTP: WTP < CV/EV
  - Higher WTA: WTA > CV/EV
  - The wedge is the commitment costs (Zhao and Kling, ’01, ’02)
Predictions

- **WTP increases**
  - As the subject is more familiar with the good
  - If she cannot delay: only chance to vote on the referendum
  - If she can’t learn much in the future
  - If she can easily reverse her vote (hard to do?)

- Predictions also form hypothetical tests
Empirical tests/evidence

- **CVM study: Corrigan, Kling and Zhao (2002)**
  - Clear lake study in Iowa
  - One group offered the opportunity of vote again one year later
  - Different levels of uncertainty (hard to manipulate)
  - Commitment cost can be 25% - 57% of static WTP (i.e. without learning)
  - WTP decreases in the option of delay
  - Responses to uncertainty somewhat weak

- **Market experiments: Kling, List and Zhao (2002)**
  - Sports card trading
  - Ask subjects’ perceptions about delay and reversal costs
  - Confirms predictions

- **Lab experiments: Corrigan (2002)**
  - Weak evidence in trading of cookies
  - Better design and more experiments are needed
Implications

- Neither WTP nor WTA may measure CV/EV accurately, if CCs are high.
- Some CCs are part of the decision, but some should be removed (esp if you want to measure the expected consumer surplus, or the value).
- Design surveys carefully to
  - Get rid of CC or OV (or estimate the magnitude)
    - More information
    - Delay vs. no delay (Hellat’s Quarry in Ames)
  - Include CC/OV to replicate the decision environment
Useful readings

- If don’t want to read the book
  - Pindyck, JEL, 1991: concise math
  - Dixit, JEP, 1992: intuition, esp. for smooth pasting

- If really want to build up the theory
  - Stokey and Lucas, 1989
  - Duffie, 1992

- If want to know the field: survey books
  - Dixit and Pindyck, 1994
  - Trigeorgis, 1996
  - Schwartz and Trigeorgis, ed., 2001

- If want more opinions from me: will put reading list online
  
  www.econ.iastate.edu/faculty/zhao